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RESEARCH INTO ADVANCED CONCEPTS OF MICROWAVE POWER AMPLIFICATION AND GENERATION UTILIZING LINEAR BEAM DEVICES

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ABSTRACT

This is an interim report which summarizes work during the past six months on a theoretical study of some aspects of the interaction between a drifting stream of electrons with transverse cyclotron motions and an electromagnetic field. Particular emphasis is given to the possible generation and amplification of millimeter waves. The report includes a discussion of the conditions for amplification, both forward wave and reverse wave, when a spiraling filamentary electron beam interacts with the TEM waves of an uniform circuit, and also gives some start-oscillation data for a reverse wave oscillator. The relativisitic origins of the amplification and oscillation are discussed.

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I. INTRODUCTION

The objective of this research program is to explore theoretically some aspects of the interaction between a drifting stream of electrons having transverse cyclotron motions and an electromagnetic field; particular emphasis being given to the possible generation and amplification of millimeter waves. Because of the interest in possible applications to millimeter wavelengths, this study concentrates on electron stream - electromagnetic field interactions which involve an uniform, or fast-wave, circuit structure.

This interim report summarizes the development of the coupled mode theory for the interaction of a spiraling filamentary electron beam with an uniform circuit with TEM waves. The conditions for amplification in both forward wave and reverse wave device configurations are discussed. It is shown that the possibility of amplification associated with the interaction between a spiraling filamentary electron beam and a TEM circuit is a consequence of the relativistic dependence of the electron mass on velocity. A limited amount of start-oscillation data for a reverse wave oscillator configuration are given. A preprint of a paper discussing spiraling filamentary electron beam interaction with fast wave circuits that was presented at the MOGA 68 meeting is included as the Appendix.

II. SPIRALING FILAMENTARY ELECTRON BEAM INTERACTION

A. Coupled Mode Theory

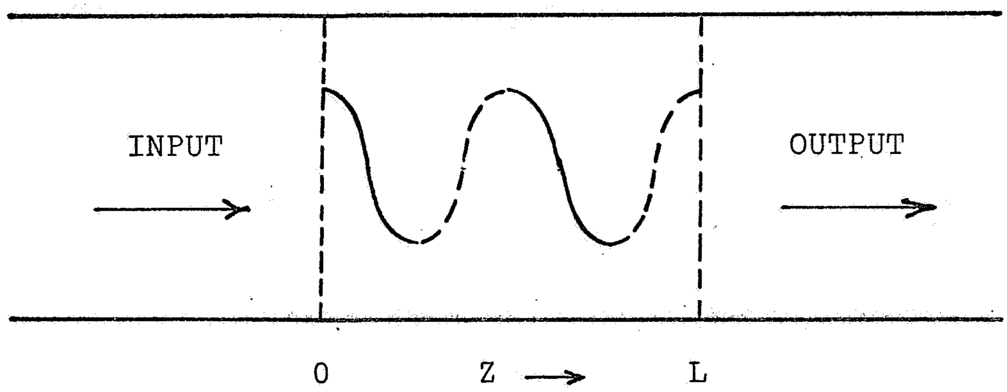
A small signal, coupled mode theory for the interaction between a spiraling filamentary electron beam and the circuit waves of an uniform circuit has been developed. Some aspects of this theory have been presented in previous Semiannual Status Reports.[1] This coupled mode theory was discussed in a paper entitled "Spiraling Electron Beam Interaction with Fast Wave Circuits" presented at the 7th International Conference on Microwave and Optical Generation and Amplification (MOGA 68) in Hamburg, Germany on September 19, 1968, and will be published in the proceedings of the conference. A preprint of this paper is ~~included~~ Detached as the Appendix for this report.

There are two differences which may be noted between the coupled mode theory discussed in the MOGA 68 paper and in the previous Semiannual Status Reports. First, an error in the development of the coupled mode theory was found and corrected for the MOGA 68 paper. This involved primarily the V wave of the spiraling filamentary electron beam. The correct development changes the $\omega - \beta$ diagram for the V beam and also allows a simplification of the coefficients of the coupled mode equations. The second difference is the introduction of more convenient normalization factors for the electron beam and circuit waves. Hereafter, the coupled mode equations used for the interaction between a spiraling filamentary electron beam and a TEM circuit will be those presented in the MOGA 68 paper (Equations (8) of the Appendix).

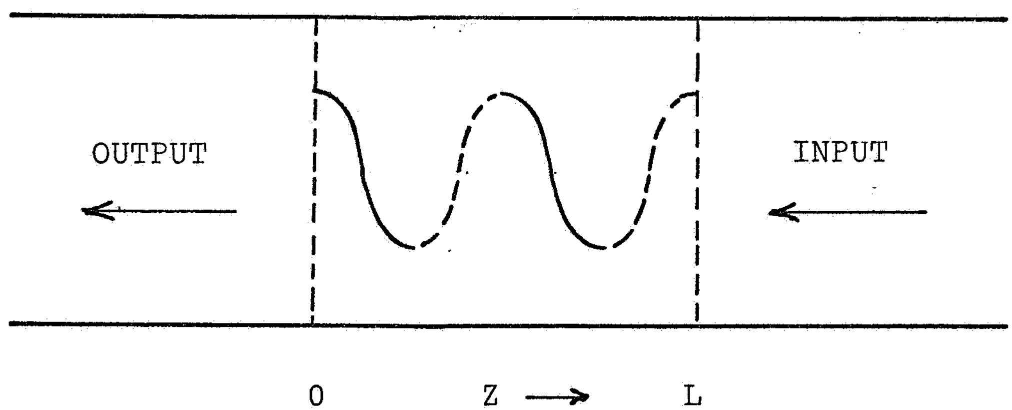
B. Amplification

The possibility of amplification resulting from the interaction between a spiraling filamentary electron beam and the TEM waves of an uniform circuit has been investigated for both a forward wave, Figure 1a, and a reverse wave, Figure 1b, interaction. In each case it was assumed that the input and output circuits were identical to the interaction circuit. Although there are a total of ten electron beam and circuit waves, only six waves couple appreciably to provide the possibility of amplification (Appendix, Section 3). These six waves are the F'_{+} and G'_{+} circuit waves and the P'_{+}, P'_{-}, V, and W beam waves.

The overall $\omega - \beta$ diagram for the coupled wave system for typical parameter values is shown in Figure 2. There are two frequency ranges which have complex phase constants and are of interest for possible amplification. These frequency bands are

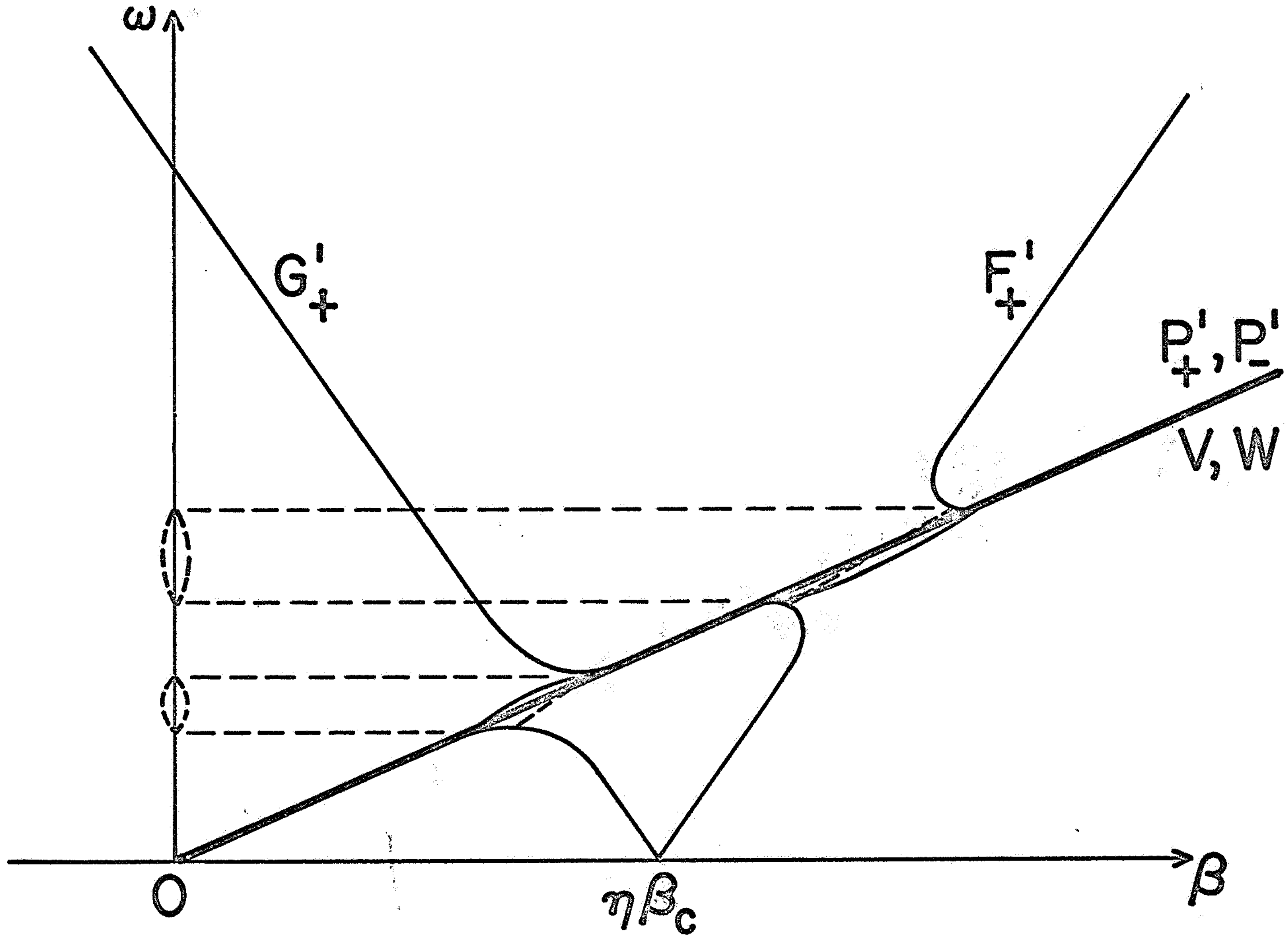


(a)



(b)

FIGURE 1. Schematic diagrams for spiraling filamentary electron beam - TEM circuit amplifier configurations. In each case the electron beam spirals from left to right in the interaction region.
(a) Forward wave amplifier.
(b) Reverse wave amplifier.



THE ω-β DIAGRAM FOR A COUPLED SPIRALING ELECTRON BEAM AND TEM CIRCUIT

FIGURE 2.

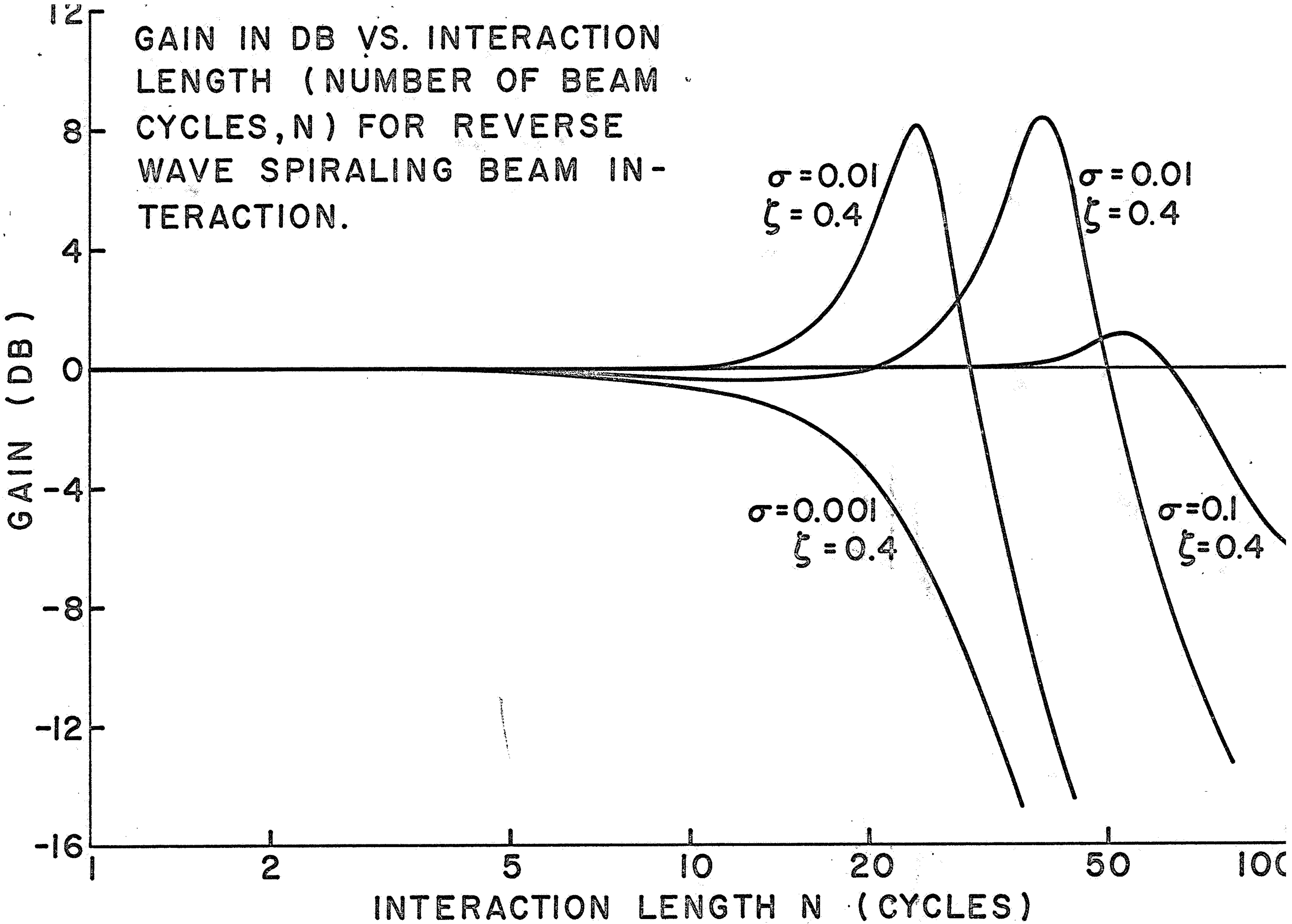
centered around frequencies on either side of the relativistic cyclotron frequency, $\eta\omega_c$; the upper frequency is $\omega_u = \eta\omega_c/(1 - \sigma)$ and the lower frequency is $\omega_l = \eta\omega_c/(1 + \sigma)$, where $\sigma = \dot{z}_o/c$.

At both frequencies, ω_u and ω_l, the possibility of amplification was explored for six wave interaction (F_+', G_+', P_+', P_-', V, and W) over the interaction length L. The boundary conditions used were the continuity of the electric and magnetic fields at z = 0, L and the absence of any electron beam modulation at z = 0. A digital computer was used to calculate the six phase constants for the coupled system, and then to calculate the output signal amplitude as a function of the interaction length L for a unit amplitude input signal.[2] For the calculations, a value of $\omega/2\pi$ = 10 GHz was assumed. In addition, the area of the TEM circuit was arbitrarily assumed to be equal to the area of a square waveguide whose cutoff frequency for the lowest mode is ω_c.

For the lower frequency of interest, ω_l, no amplification was found for either the forward wave or the reverse wave configurations. A wide range of values of the longitudinal velocity parameter $\sigma = \dot{z}_o/c$ and the rotational velocity parameter $\zeta = \omega_c r_o/c$ were explored. As a consequence, it is believed that under no circumstances would gain be found for spiraling beam interaction with a TEM circuit in the neighborhood of ω_1.

However, for the upper frequency of interest, ω_u, gain was found for both the forward wave and the reverse wave configurations over a range of values for σ and ζ. The more critical parameter appears to be σ; a value of $\sigma = 0.01$ gave gain, while values of 0.001 and 0.10 gave no gain. Figure 3 shows the gain versus interaction length for several values of σ and ζ for the reverse wave configuration with a matched load. Figure 4 gives a typical result for the forward wave configuration with a matched load. In both of these figures the interaction length is measured in terms of N, the number of cycles the d-c spiraling filamentary electron beam makes in the interaction region.

It is interesting to note for the reverse wave configuration, Figure 3, that the gain (for parameter values for which it exists) has a single maximum as a function of N and falls off rapidly for large values of N. On the other hand, for the forward wave configuration, the gain has several maxima in the range of N explored. In this case, the gain exhibits some of the characteristics of a beating wave phenomenon. Clearly, however, the amplification for the spiraling



GAIN IN DB VS. INTERACTION LENGTH (NUMBER OF BEAM CYCLES, N) FOR REVERSE WAVE SPIRALING BEAM INTERACTION.
$\sigma = 0.01$
$\zeta = 0.4$
$\sigma = 0.01$
$\zeta = 0.4$
$\sigma = 0.001$
$\zeta = 0.4$
$\sigma = 0.1$
$\zeta = 0.4$
GAIN (DB)
12
8
4
0
-4
-8
-12
-16
1
2
5
10
20
50
100
INTERACTION LENGTH N (CYCLES)

FIGURE 3.

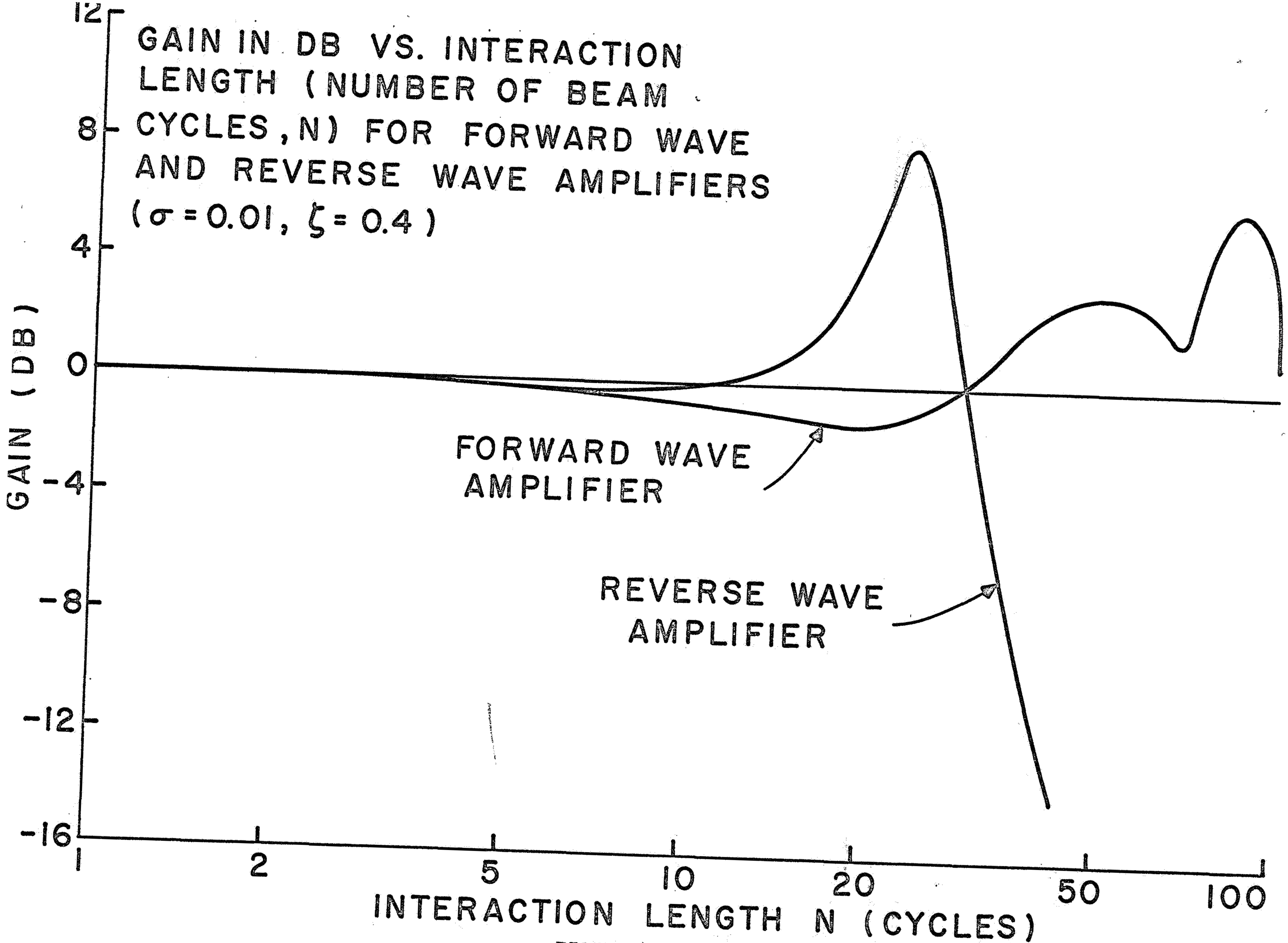

GAIN IN DB VS. INTERACTION LENGTH (NUMBER OF BEAM CYCLES, N) FOR FORWARD WAVE AND REVERSE WAVE AMPLIFIERS
(σ = 0.01, ζ = 0.4)
GAIN (DB)
12
8
4
0
-4
-8
-12
-16
1
2
5
10
20
50
100
INTERACTION LENGTH N (CYCLES)
FORWARD WAVE AMPLIFIER
REVERSE WAVE AMPLIFIER

FIGURE 4.

filamentary electron beam interaction with a TEM circuit has a different behavior than does the amplification in a conventional traveling wave tube.

The initial gain calculations were all made assuming a matched load for the amplifier. The effects of a mismatched load on the amplification were briefly explored for the reverse wave amplifier with $\sigma = 0.01$, $\zeta = 0.40$. Using the optimum interaction length of $N = 24$ for the matched load case, the gain was computed for four values of load mismatch; the results are shown in Table 1. It is seen that the gain can be increased by using a load which is not matched. The optimum load impedance was not sought, since the interaction is of more interest for possible application to oscillators than to amplifiers.

$\frac{Z_L}{Z_o}$	Gain, db
1.0	8.17
1.1	8.77
0.9	7.38
1.0 + j0.1	9.73
1.0 - j0.1	6.80

Table 1. Reverse wave amplification for a mismatched load.

C. Relativistic Origin of the Amplification

The question of the role that relativistic effects play in the amplification produced by the interaction between a spiraling filamentary electron beam and a TEM circuit has been explored. This has been accomplished by developing the coupled mode theory while neglecting relativistic effects; this theory was then compared to the previous coupled mode theory which included relativistic effects. The non-relativistic expressions generally follow from the relativistic expressions by setting $\eta = 1.0$ and $\sigma = \zeta = 0$ (although the latter is not strictly correct in all cases).

The major results for the non-relativistic case are the beam wave definitions

$$P_+' = C(u_+' + u_-'), \tag{1a}$$

$$P_-' = C(u_+' - u_-'), \tag{1b}$$

$$Q_+' = C(u_+' - j\omega_c r_+'), \tag{1c}$$

$$Q_-' = C(u_-' + j\omega_c r_-'), \tag{1d}$$

$$V = 2C\dot{z}_1/\zeta, \tag{1e}$$

$$W = 2C\omega_c z_1/\zeta, \tag{1f}$$

(the circuit wave definitions are unchanged), and the coupled wave equations

$$\left(\frac{\partial}{\partial z} + j\beta_e\right)P_+' + K[(1 -\sigma)(F_+' + F_-')+(1 +\sigma)(G_+' + G_-')] = 0, \tag{2a}$$

$$(\frac{\partial}{\partial z} + j\beta_e)P_-' + K[(1 - \sigma)(F_+' - F_-')+(1 + \sigma)(G_+' - G_-')] = 0, \tag{2b}$$

$$(\frac{\partial}{\partial z} + j\beta_e + j\beta_c)Q_+' + K[(1 - \sigma)F_+' + (1 + \sigma)G_+'] = 0, \tag{2c}$$

$$(\frac{\partial}{\partial z} + j\beta_e - j\beta_c)Q_-' + K[(1 - \sigma)F_-' + (1 + \sigma)G_-'] = 0, \tag{2d}$$

$$(\frac{\partial}{\partial z} + j\beta_e)V - jK(F_+' - F_-' - G_+' + G_-') = 0, \tag{2e}$$

$$(\frac{\partial}{\partial z} + j\beta_e)W - \beta_c V = 0, \tag{2f}$$

$$(\frac{\partial}{\partial z} + jk + j\beta_c)F_+' - K(P_+' + P_-') = 0, \tag{2g}$$

$$(\frac{\partial}{\partial z} + jk - j\beta_c)F_-' - K(P_+' - P_-') = 0, \tag{2h}$$

$$(\frac{\partial}{\partial z} - jk + j\beta_c)G_+' + K(P_+' + P_-') = 0, \tag{2i}$$

$$(\frac{\partial}{\partial z} - jk - j\beta_c)G_-' + K(P_+' - P_-') = 0. \tag{2j}$$

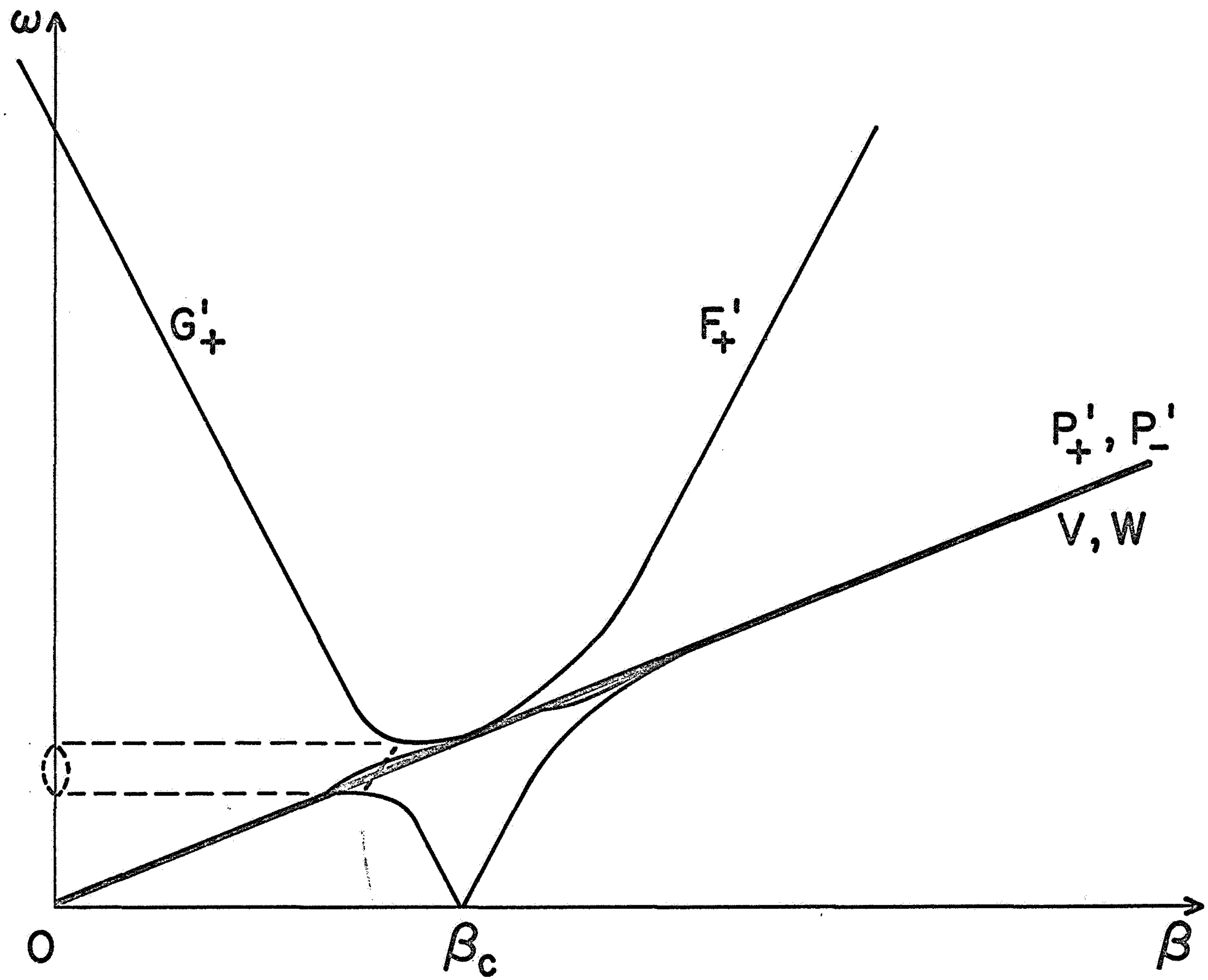
The terms in σ which appear in the first four of the coupled wave equations, (2a) - (2d), arise from the magnetic field terms in the force equation. These might be considered terms of relativistic order and neglected, but the conclusions to be given below would be unchanged.

There are two main differences between the relativistic and non-relativistic expressions. First, the P_+' and P_-' beam waves are not coupled in the absence of a circuit wave in the non-relativistic theory (compare Equation (2a) above with Equation (8a) of the Appendix).

Thus the weak instability of the spiraling filamentary electron beam in the absence of a circuit wave (see Equations (10a) and (10b) of the Appendix) is a relativistic effect.

Second, comparing Equations (8g) to (8j) of the Appendix with Equations (2g) to (2j) above, it is seen that in the non-relativistic theory the V beam wave is not coupled into the circuit wave equations, while it is in the relativistic theory. This coupling of the V beam wave to the circuit waves is believed to be one of the basic elements of the amplification mechanism and is discussed further below.

The overall $\omega - \beta$ diagram for the non-relativistic coupled wave system is shown in Figure 5. Comparing this $\omega - \beta$ diagram with the $\omega - \beta$ diagram for the relativistic coupled wave theory, Figure 2, the main difference is seen to lie in the character of the phase constants in the neighborhood of ω_u. Whereas the relativistic theory gives a complex conjugate pair of values in addition to a set of real values for β, the non-relativistic theory yields a set of values for β which are all real. Because of this difference in the character of the values for β in the neighborhood of ω_u, the non-relativistic theory predicts that no amplification will be found for the interaction of a



THE ω-β DIAGRAM FOR A COUPLED SPIRALING ELECTRON BEAM AND TEM CIRCUIT (NON-RELATIVISTIC THEORY)

FIGURE 5.

spiraling filamentary electron beam with a TEM circuit.

Thus, we conclude that the occurrence of amplification in the neighborhood of ω_u when a spiraling filamentary electron beam interacts with a TEM circuit is a relativistic effect. It is interesting to note the profound influence on the possibility of amplification produced by the relativistic effect. And this is true even though the actual velocities involved need not be in the range normally considered relativistic.

The basic mechanism which produces the amplification is believed to be the coupling between the V beam wave and the circuit waves. This coupling is caused by the relativistic dependence of the electrons' mass on their velocity, so that their transverse motion is coupled to their longitudinal motion. Since the transverse motion is coupled to the circuit waves, the longitudinal motion is also. The coupling of the V beam wave to the other waves provides a mechanism for the transfer of d-c axial kinetic energy of the electrons into a-c energy distributed among the various waves of the coupled system in the neighborhood of ω_u. This leads to the possibility of amplification for the spiraling filamentary electron beam - TEM circuit system. The fact that several waves can interact strongly means that the details of the interaction are complex and not easily described.

D. Oscillation

The possibility of oscillation in a spiraling filamentary electron beam - TEM circuit system has been investigated; in particular, the reverse wave oscillator configuration of Figure 6 for frequencies in the neighborhood of ω_u. The procedure followed was to consider the device as a reverse wave amplifier with a unit amplitude input signal and use a digital computer to calculate the gain for various parameter values, searching for a set of values which gave infinite gain. This set of parameter values gives the start-oscillation conditions for the device.

The velocity parameters were held fixed at $\sigma = 0.01$, $\zeta = 0.20$, and three values of the d-c beam current, $I_o = 0.01$, 0.10, and 1.0 amperes, have been explored. The parameters varied to establish the start-oscillation conditions for each value of the d-c beam current were the start-oscillation interaction length N_s (measured in terms of the number of d-c beam cycles) and the ratio of the start-oscillation frequency ω_s to $\omega_u = \eta\omega_c/(1 - \sigma)$. The numerical results for the start-oscillation conditions for the reverse wave

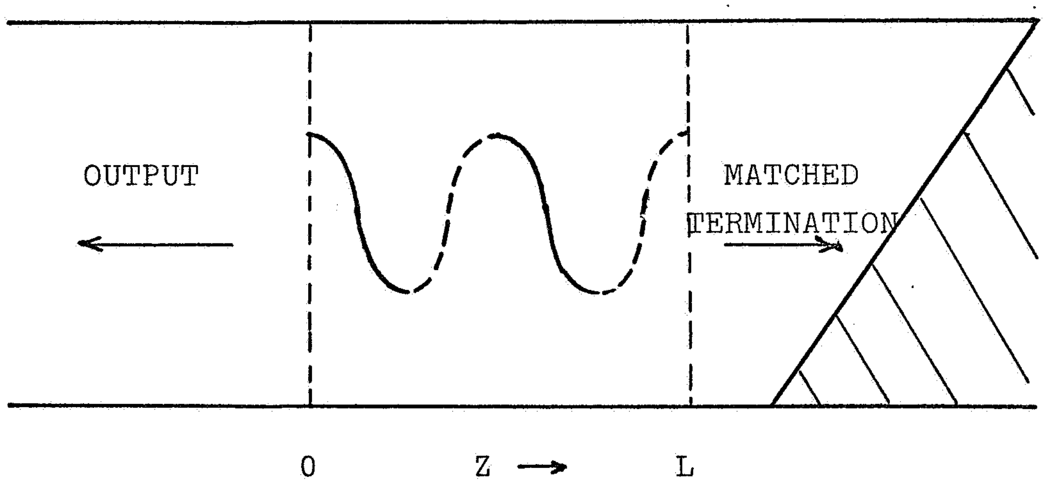


FIGURE 6. Reverse wave oscillator configuration for spiraling filamentary electron beam interaction with a TEM circuit.

oscillator are given in Table 2.

D-C Beam Current I_o, amp.	Start-Oscillation Frequency, ω_s/ω_u	Start-Oscillation Length, N_s, Cycles	Normalized Start-Oscillation Length, KL_s
0.01	0.9980	238	0.652
0.10	0.9966	85.1	0.736
1.0	0.9969	33.8	0.924

Table 2. Start-oscillation conditions for a reverse wave oscillator (σ = 0.01, ζ = 0.20).

Although the data available are limited, some conclusions may be drawn from the start-oscillation conditions given in Table 2. First, the start-oscillation length measured in d-c beam cycles, N_s, decreases with increasing d-c beam current I_o. Based on the three data points available, approximately

$$N_s \cong 33.8\ I_o^{-0.40}. \tag{3}$$

A normalized start-oscillation length KL_s is given in the fourth column of Table 2. Here, K is the coupling factor for the interaction system (K is defined following Equations (8) of the Appendix), L_s is the start-oscillation length in meters, and the product is dimensionless. Based on the three data points,

$$KL_s \cong 0.924\ I_o^{0.10}. \tag{4}$$

Thus KL_s is seen to be a very slowly varying function of the d-c beam current I_o.

The start-oscillation frequency is seen to lie very close to ω_u; within 0.05% in the three cases considered. The dependence on the d-c beam current does not follow a simple pattern, which is perhaps not surprising since a relatively complex six wave interaction occurs.

From the discussion of the preceding section, it is clear that the occurrence of oscillations is produced by the relativistic dependence of the electrons' mass on their velocity. Without this relativistic effect, the interaction of a spiraling filamentary electron beam with a TEM circuit would not produce oscillations. It is seen that this coupled mode theory is effective in establishing the start-oscillation conditions for spiraling filamentary electron beam interaction with fast wave circuits. In the future, this theory will be applied to the more practical case of spiraling filamentary electron beam interaction with the TE modes of a square or circular waveguide.

References

1. P. R. McIsaac, Semiannual Status Reports, NASA Research Grant NGR 33-010-047, December 1967 and June 1968.

2. J. F. Rowe, Jr., "A Study of the TEM Wave-Spiraling Electron Beam Amplifier," M.S. Thesis, Cornell University, February 1969.